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ON A THEOREM OF LAMBERT'S.

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JOHN HENRY LAMBERT was born at Mülhausen, in Alsace, Aug. 26, 1728, and died at Berlin, Sept. 25, 1777. Lambert's father, a French refugee, was a tailor, and on account of poverty and a large family of children, he was able to give his son but little assistance in his early studies. Lambert, however, by means of his own wonderful industry and application became one of the most accomplished philosophers of his time. He was the author of many scientific and mathematical papers, and among the most important of his works may be mentioned the following:

(1). Photometria, sive de mensura et gradibus luminis, colorum, et umbræ. Augsburg, 1760.

(2). Insigniores Orbitæ Cometarum Proprietates. Augsburg, 1761.

(3). The establishment of the Astronomisches Jahrbuch. Berlin, 1774.

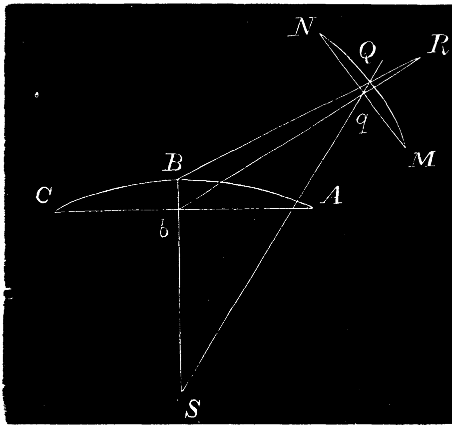
In the second of these works, Lambert gives the elegant theorem now known by his name and in common use for computing the orbit of a comet. This theorem had indeed been proven for the parabola by Euler in 1740, but it had been forgotton, and was re-discovered by Lambert and extended by him to the ellipse and hyperbola. Another formula given by Lambert in this work has been used by Encke in his excellent discussion of Olber's Method for computing the orbit of a comet. This formula gives the relation between the chord, the sum of the radii vectores and the time, by means of a rapidly converging series; and by tabulating this series for different values of the variable, and by giving a proper form to the computation, Encke has reduced the solution of Lamberts Equation by trial to a sure methodical process.

In the Memoirs of the Berlin Academy 1771, p. 352, Lambert gives a theorem on the apparent orbit of a comet which is interesting. This theorem may be stated as follows:

"Take two not very distant points on the apparent orbit of a comet, and draw through these points an arc of a great circle. If the apparent orbit between these points turns from this great circle toward the places of the Sun for the intermediate times, the comet is farther from the Sun than the Earth is, and in the other case, it is nearer. If the comet moves in a great circle, the comet and the Earth are at the same distance from the Sun."

This theorem is indeed of no great practical importance, since as soon as we know the real orbit of a comet we can compute its apparent geocentric path, but it gives us in a very simple manner some idea of the distance of

a comet. In his Mécanique Céleste, Tome I., p. 208, Laplace has given an analytical proof of this theorem, but it is interesting to consider the simple geometrical method by which Lambert establishes it.



Let S be the place of the Sun, MQN a small arc of the orbit of the comet, and ABC the corresponding places of the Earth in its orbit. Let Q and B be the middle points of the arcs MN and AC and draw the lines as in the figure. The curvature of the arcs MN and AC being a result of gravity, we shall have very nearly

$$qQ \ : \ bB :: \frac{k}{SQ^2} : \frac{k}{SB^2},$$

k being a constant depending on the attractive force of the Sun. If we suppose the Earth and comet to describe the chords AbC, MqN with uniform velocities the apparent orbit of the comet will be a great circle of the sphere. In the case of the actual orbits when the Earth is at B, the comet is seen at Q, but in the case of the rectilinear orbits when the Earth is at b the comet is seen at q. If the lines BQ and bq are parallel, these apparent places are the same. Since the points b and q are in the lines SB and SQ, if we suppose BQ parallel to bq, we have

$$Bb \ : \ Qq :: SB \ : \ SQ,$$

and hence, by means of the first proportion,

$$SQ = SB.$$

Thus when the right lines BQ and bq are parallel, the comet and the Earth are at the same distance from the Sun.

If we now suppose SQ greater than SB, then Bb will be greater than Qq, and the right lines BQ and bq intersect at a point R, beyond the comet. The angle SBR is smaller than the angle SbR, and the apparent place of the comet seen from the Earth is less removed from the Sun than when seen from the point b, or than in the case of its motion on a great circle.

Again, suppose SQ less than SB, then Bb is smaller than Qq, and the angle SBQ will be greater than the angle Sbq, or the apparent place of the comet will be more removed from the Sun than in the case of its motion on a great circle. If the comet moved exactly in the plane of the Ecliptic, its apparent orbit would be a great circle, but such an exceptional case has never been known to occur.

It will be remembered that a century ago the problem of computing the orbit of a comet was much discussed by mathematicians and astronomers, and almost every eminent man tried his hand on this problem. Kepler discussed this question, and assumed that the comets moved in right lines; and as he never wanted for an hypothesis, he also assumed that these erratic bodies were generated in the celestial spaces, and then were destroyed in some mysterious manner. Newton's discovery of the law of gravitation showed that the rectilinear motion of a comet was impossible, but this motion continued to be assumed for small portions of the orbit, and was used by Newton himself as a first approximation. It is worthy of notice that in the memoir refered to above, Lambert divides the chord *MN* at the point *q* in such a manner that the segments of this chord are proportional to the intervals of time between the observations. This assumption was afterward adopted by Olbers in his method of computing the orbit of a comet, a method which is now almost the only one in use.

AN ACCOUNT OF CAUCHY'S "CALCUL DES RESIDUS."

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(*Continued from page* 46.)

§ 6.

THERE are other applications, more or less direct, of which a few examples may be given.

Ex. 6. Assume in (6) $f(z) = e^{-c^2z^2} = e^{-c^2(x+yi)^2} = e^{-c^2(x^2-y^2)}(\cos 2c^2xy - i \sin 2c^2xy)$.

Since $e^{-c^2z^2} = \infty$ if $z = 0 \pm \infty i$ there would be a point of discontinuity if 0 was within the x-integration and $\pm\infty$ the upper or lower limit of the y-integration. But the correction would be 0 nevertheless for we have by $(10''_3)$

$$\Delta_{0\pm\infty i} = \pi i\left[\frac{z\mp\infty i}{e^{c^2z^2}}\right]_{z=\pm\infty i} = \pi i\left[\frac{1}{2c^2z\,e^{c^2z^2}}\right]_{z=\pm\infty i} = 0.$$

There is consequently no correction, and we have by (6)

$$\int_{x_0}^{x_n} dx\left[e^{-c^2(x^2-y_n^2)}(\cos 2c^2xy_n - i\sin 2c^2xy_n) - e^{-c^2(x^2-y_0^2)}(\cos 2c^2xy_0 - i\sin 2c^2xy_0)\right]$$
$$= i\int_{y_0}^{y_n} dy\left[e^{-c^2(x_n^2-y^2)}(\cos 2c^2x_ny - i\sin 2c^2x_ny) - e^{-c^2(x_0^2-y^2)}(\cos 2c^2x_0y - i\sin 2c^2x_0y)\right]. \qquad (g)$$

Let $x_0 = 0 = y_0$, then